Aarhat Multidisciplinary International Education Research Journal (AMIERJ)
ISSN 2278-5655
Impact Factor : 0.948

Bi-Monthly

Vol - II  Issues - V  [2013-14]

Chief Editor:
Ubale Amol Baban

Editorial/Head Office: 108, Gokuldham Society, Dr.Ambedkar chowk, Near TV Towar,Badlapur, MS
INEQUALITIES ON NUMBER THEORETIC FUNCTIONS

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Abstract
We have observed the inequalities on number theoretic functions and one can easily find out the maximum and minimum fluctuations in a defined range.

1. Introduction:
The theory of numbers in that branch of Mathematics which deals with properties of whole number, 1, 2, 3, 4, 5, 6,………………….. also called the counting numbers, or positive integers. The positive integers are undoubtedly man’s first mathematical creation. It is hardly possible to imagine human beings without the ability to count, at least within a limited range. Historical record shows that’s early as 5700 B.C., the ancient Sumerians kept a Calendar, so they must have developed some from arithmetic.

When ancient civilizations reached a level which provided leisure time to ponder about things, some people begin to speculate about the nature and properties of numbers. This curiosity developed in to a sort of number mysticism or numerology, and even today numbers such as 3, 7, 11, and 13 are considered omens of good or bad luck.

The professional mathematician is attracted to number theory because the way all the weapons of modern mathematics can brought to bear on its problems. As a matter of fact, many important branches of mathematics had their origin in number theory. New problem arise more rapidly and many of the old problems have remained unsolved for centuries.

2. Formulations:
\[ \Delta d(x) = d(x) - d(x + 1), \quad x \geq 1 \]  
\[ \Delta \mu(x) = \mu(x) - \mu(x+1), \quad x \geq 1 \]  
\[ \Delta \varphi(x) = \varphi(x) - \varphi(x+1), \quad x \geq 1 \]

Where \( d(x) \) denotes divisor function:
Where \( \mu(x) \) denotes mobius function:
Where \( \varphi(x) \) denotes Euler's totient function:
\[ \Delta \sigma(x) = \sigma(x) - \sigma(x+1), \quad x \geq 1 \] 

.............2.4

Where \( \sigma(x) \) denotes sum of the divisors of \( x \):

3. Experiment:

For the numbers 1 to 100 a table has been formulated for (2.1) and following results have been obtained:

4. Theorem: With the help of table 4 (c)

If \( \Delta d = k \) then

\[ \prod_{i=1}^{p} (\alpha_i + 1) = \prod_{j=1}^{m} (\beta_j + 1) + k \]

Where \( x = \prod_{i=1}^{p} p_i^{\alpha_i} \) and \( x + 1 = \prod_{j=1}^{m} q_j^{\beta_j} \)

In particular

If \( K = 0 \) then \( \Delta d = 0 \)

\( \Rightarrow x, x+1 \) have similar factorization

Proof can be easily seen with the help of table 4(c)

5. Experiment: For the numbers 1 to 100 a table has been formulated for (2.2) and following results have been obtained:

Theorem 5.1:

<table>
<thead>
<tr>
<th>Value</th>
<th>Behaviour for ( x, x+1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_i = 1 \forall i ) &lt;br&gt;( K = \text{odd} )</td>
<td>( \beta_j = 1 \forall j ) &lt;br&gt;( I = \text{even} )</td>
</tr>
<tr>
<td>( \alpha_i = 1 \forall i ) &lt;br&gt;( K = \text{odd} )</td>
<td>For at least one value of ( j, \beta_j &gt; 1 )</td>
</tr>
<tr>
<td>( \alpha_i &gt; 1 ) &lt;br&gt;( K = \text{odd} )</td>
<td>( \beta_j = 1 \forall j ) &lt;br&gt;( I = \text{even} )</td>
</tr>
</tbody>
</table>
\[ \mu(x) = \begin{cases} 
0 & \text{For at least one value of } i, \alpha_i > 1 \quad \text{For at least one value of } j, \beta_j > 1 \\
\alpha_i = 1 \forall i & \beta_j = 1 \forall j \\
K = \text{odd} & l = \text{odd} 
\end{cases} \]

\[ \begin{array}{c|c|c}
\beta_j = 1 \forall j & \text{For at least one value of } j, \beta_j > 1 \\
l = \text{even} & \alpha_i = 1 \forall i \\
K = \text{odd} & 
\end{array} \]

\[ \begin{array}{c|c|c}
2 & \beta_j = 1 \forall j & \text{For at least one value of } j, \beta_j > 1 \\
l = \text{even} & \alpha_i = 1 \forall i & K = \text{odd} 
\end{array} \]

**Proof** can be easily seen with the help of table 4(d).

6. **Experiment:** For the number 1 to 100 a table has been formulated for 2.3 and following results have been obtained.

<table>
<thead>
<tr>
<th>Group Size</th>
<th>Maximum fluctuation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>50</td>
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<tr>
<td>100</td>
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</tr>
</tbody>
</table>

Table 4(a)

**Theorem 7.0** If \( \varphi \) denotes Euler's function then \( \Delta \varphi \) is increasing the interval.

**Proof** can be easily seen with the help of table 4(a) and 4(f).

7.1 **Experiment:** With the help of table 4(h) for \( x \) (1 to 200) by taking the group size as divisor of \( x \) (e.g. 200) 1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 200 following observations have been made:
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</table>

Table 4(b)

**Theorem 8.0** If $\sigma$ denotes sum of the divisors then $\Delta \sigma$ have maximum fluctuation at group size one and remains constant in other intervals.

**Proof** can be easily seen with the help of tables 4(b), 4(g) and 4(h).

\[
d(x) = \sum \frac{1}{d(x)}
\]

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<th>d(x)</th>
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Table 4(c)
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Table 4 (d)
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Table 4(e)

$\Delta \varphi (x)$
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Table 4 (h)

9. REFERENCE