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STUDY OF MONOTONIC FUNCTIONS

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Abstract:

This paper is designed to give readers a basic understanding on the topic of monotonic function. In mathematics monotonic function is a function between ordered sets that preserves or a reverses the given order. This concept first arrows in calculus and was letter A generalized to the more abstract setting of order theory.

Keywords: *Monotonic , monotonicity, global Maximum , global minimum*

WHAT IS MONOTONIC FUNCTION?

A monotonic function is a function which is either $\{M_n\}$ entirely no increasing or non-decreasing. A function is monotonic if its first derivative (which need not be continuous) does not change sign. The term monotonic may also be used to describe set-functions which map subsets of the domain to non-decreasing values of the codomain. In particular, if $f: X \rightarrow Y$ is a set function from a collection of sets X to an ordered set Y, then f is said to be monotone if whenever $A \subseteq B$ as elements of X, $f(A) \leq f(B)$. This particular definition comes up frequently in measure theory where many of the families of functions defined (including outer measure, premeasure, and measure) begin by considering monotonic set functions.

MONOTONICITY

The **monotonicity** of a function tells us if the function is increasing or decreasing. A function is **increasing** when its graph rises from left to right. In technical terms, a function is increasing on an interval I if for any x_1 and x_2 in I, $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$.

Basically, this says that when $x_1 < x_2$, then the function evaluated at $x1$ is less than the function evaluated at x_2.

A function is decreasing when its graph falls from left to right. Again, the technical definition says that a function is decreasing on an interval I if for any x_1 and x_2 in I, $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$.

DEFINITION OF MONOTONIC FUNCTION

When a function is increasing on its entire domain or decreasing on its entire domain, we say that the function is **strictly monotonic**, and we call it a **monotonic function.**

How To Check Monotonic function

The function is monotonically increasing if first derivative of f(x),f1(x) >= 0.

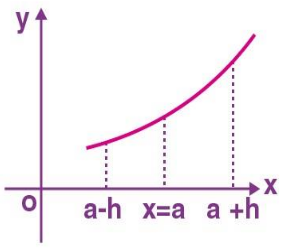
The function is monotonically decreasing if first derivative of f(x),f1(x) <= 0.

The function is monotonically constant if first derivative of f(x),f1(x) = 0.

Monotonicity of a function at a point

A function is said to be monotonically increasing at x = a if it satisfies f(a + h) > f(a) f(a – h) < f(a)

Where h is very small value



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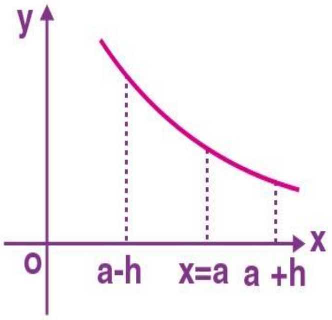
A function is said to be monotonically decreasing at x = a if it satisfies f(a + h) < f(a) f(a – h) > f(a)
Where h is very small value



Examples on Monotonicity and Extremum of functions:

Example 1: Prove that f(x) = x – sin(x) is an increasing function.

Solution:

f(x) = x – sin(x)

⇒ dy/dx = 1 – cos(x)

dy/dx ≥ 0 as cos(x) having value in interval [-1,1] and dy/dx = 0 for the discrete values of x and do not form an interval, hence we can include this function in monotonically increasing function.

Example 2: Find the interval of monotonicity for f(x) = x/(log(x)).

Solution:

f(x) = x/(log(x))

⇒ dy/dx = (log(x) – 1)/$(\log(x))^2$ dy/dx > 0 log x – 1 > 0 ⇒ x > e f(x) is increasing for x> e.

dy/dx < 0

⇒ log x – 1 < 0 ⇒ x < e

f(x) is decreasing for x< e.

Maxima And Minima

Local Maxima is the point on the graph or the curve where the value of the function is higher than the limiting function value.

Local Minima is the point on the graph or the curve where the value of the function is lower than the limiting function value.

Global Maxima is the point on the graph or curve that is the maximum and the highest value of the function among different numbers of critical points in the function.

Global Minima is the point on the graph or curve that is the minimum and least value of the function among different numbers of critical points in the function.

A function f(x) is said to obtain a maximum at x = p if there exists a neighbouring point within the close proximity of other x elements, so that (p – θ, p + θ).

Example:

Q1. Prove that y = f(x) = x – cos(x) is an increasing function by the help of monotonicity. Solution: f(x) = x – cos(x) => dy/dx = 1 + sin(x) dy/dx will always be greater than zero because sin(x) having values within the interval [-1,1] and dy/dx = 0, hence we can include this function in a monotonically increasing function.

Conclusion

A monotonic function does help in aiding the simplification to get to the depth of limits and neighbouring elements. A function can have a multiple number of local maxima and local minima at multiple points but there is only a single global maximum or single global minimum. The value of these local maxima and local minima does not necessarily has to do anything with the global maxima and global maximum. This article showed us the importance of the monotonicity of a function, its importance at the time of studying the continuity and derivatives of the functions.

Monotonic function in calculus and analysis:

In calculus, a function defined on a subset of the real numbers with real values is called monotonic if and only if it is either entirely non-increasing, or entirely non-decreasing. That is, as per Fig. 1, a function that increases monotonically does not exclusively have to increase, it simply must not decrease.

A function is called monotonically increasing (also increasing or nondecreasing)

[3] if for all and such that $x \leq y$ one has $f(x) \leq f(y)$, so preserves the order (see Figure 1). Likewise, a function is called In calculus and analysis monotonically decreasing (also decreasing or non-increasing) if, whenever $x \leq y, then\ (x) \geq f(y)$, then , so it reverses the order (see Figure 2).

If the order $\leq$ in the definition of monotonicity is replaced by the strict order $\leq$, one obtains a stronger requirement. A function with this property is called strictly increasing (also increasing). Again, by inverting the order symbol, one finds a corresponding concept called strictly decreasing (also decreasing). A function with either property is called strictly monotone. Functions that are strictly monotone are one-to-one (because for x not equal to y, either $x < y$ or $x > y$ and so, by monotonicity, either $f(x) < f(y)$ or $f(x) > f(y)$, thus $f(x) \neq f(y)$.)

Inverse of function:

All strictly monotonic functions are invertible because they are guaranteed to have a one-to-one mapping from their range to their domain.

However, functions that are only weakly monotone are not invertible because they are constant on some interval (and therefore are not one-to-one).

A function may be strictly monotonic over a limited a range of values and thus have an inverse on that range even though it is not strictly monotonic everywhere. For example, if $y = (x)$ is strictly increasing on the range $[a, b]$, then it has an inverse $x = h(y)$ on the range $[g(a), g(b)]$.

MONOTONIC SEQUENCE:

A sequence $\{S_n\}$ is said to be a monotonic increasing, if $S_{n+1} \geq S_n\ \forall$ n, and monotonic decreasing if $S_{n+1} \leq S_n\ \forall$ n. It is said to be monotonic if it is either monotonic increasing or monotonic decreasing.

❖ **Theorem:**

A necessary and sufficient condition for the convergence of a monotonic sequence is that it is bounded.

Proof: The condition is necessary for we know that every convergence sequence is bounded.

The condition is sufficient. Let a bounded sequence $\{S_n\}$ be monotonic increasing. Let S denotes its range, which is evidently bounded. By the completeness property, S has the least upper bound (the supremum), say M.

We shall show that $\{S_n\}$ converges to M.

Let ε be any pre-assigned positive number.

Now since M- ε is a number less than the supremum M, there exist at least one member say S_m such that $S_m >$ M- ε.

As $\{S_n\}$ is a monotonic increasing sequence,

∴ $S_n \geq S_m > M - \varepsilon, \quad \forall n \geq m$ Again, since M is supremum,

$S_n \leq M < M + \varepsilon, \quad \forall n \geq m$

Thus

$M - \varepsilon < S_n < M + \varepsilon, \quad \forall n \geq m$

$\Rightarrow \quad |S_n - M| < \varepsilon, \quad \forall n \geq m$

$\Rightarrow \{S_n\}\ converges\ and\ \lim Sn = M.$

We may similarly consider the case of a bounded monotonic decreasing.

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SUBSEQUENCES :

If $\{S_n\} = \{S_1, S_2, S_3, \ldots\ldots\}$ be a sequence, then any infinite succession of its terms, picked out in any way (but preserving the original order), is called a subsequence of $\{S_n\}$.

ILLUSTRATIONS

1. $\{S_2, S_4, S_6, \ldots\ldots, S_{2n}, \ldots\ldots\}$ is a subsequence of $\{S_n\}$.
2. $\{S_1, S_3, S_5, \ldots\ldots, S_n, \ldots\ldots\}$ is a subsequence of $\{Sn\}$.
3. $\{S_7, S_8, S_9, \ldots\ldots\}$ is a subsequence of $\{Sn\}$, which is obtained by removing a finite number of terms from the beginning of $\{S_n\}$.

Example 1. Show that the sequence $\{S\}$, where

$$S_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+n}, \quad \forall\, n \in N$$

Is convergent.

✦ Now

$$S_{n+1} - Sn = \frac{1}{2n+1} \quad \frac{1}{2n+2} \quad \frac{1}{n+1} + \quad -$$

$$= \frac{1}{2(n+1)(2n+1)} > o, \quad \forall\, n$$

∴ The sequence $\{S\}$ is monotonic increasing.

Again

$$Sn = \frac{1}{2n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+n}$$

$$< \frac{1}{n} + \frac{1}{n} + \cdots + \frac{1}{n} = 1$$

i.e.,

$0 < S_n < 1$

∴ The sequence is bounded.

Hence, the sequence being bounded and monotonic increasing, is convergent.

❖ **Theorem:**

Cantors' intersection for real line. *If $F = \{F_n\}$ is a countable class of nonempty closed and bounded set such that*

$F_1 \supset F_2 \supset f_{3\ldots\ldots} \supset F_n$ then $\bigcap_{n=1}^{\infty} F$ is non-empty.

Since each F_n is anon empty closed and bounded set therefore there exit sequence of real member M and m_n belonging to F_n such that

$$M_n = \sup F_{n1} m_n = inf\ F_n$$

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Then $M_n > M_{n+1}$ and $m_n \leq m_{n+1}$ for each $n \in N$. Now the lower bound for the set $\bigcap_{n=1}^{\infty} F_n$ is the lower bound for the sequence $\{M_n\}$ of upper bounds.

Thus $\{M_n\}$ is a non-increasing sequence which is bounded below and therefore convergent.

Let $\lim_{n\to\infty} M_n = M$

We shall show that $\in \bigcap_{n=1}^{\infty} F_n$. Let, if possible, $M \notin \bigcap_{n=1}^{\infty} F_n$. Then there will be at least one neighbourhood, say, $]M - \varepsilon, M + \varepsilon[$, $\varepsilon > 0$ which contains no point of $\bigcap_{n=1}^{\infty} F_n$

$\Rightarrow$ $]M - \varepsilon, M + \varepsilon[$ contains no point of F_n for some value of n, say, m.

$\Rightarrow$ $]M - \varepsilon, M + \varepsilon[$ contains no point of F_n, for $n \geq m$

$\Rightarrow M_n \notin]M - \varepsilon, M + \varepsilon[, \quad \forall n \geq m,$

Contradicting the fact that $\{M_n\}$ converges to M.

Hence, $M \in \bigcap_{n=1}^{\infty} F_n$.

REFERENCES:

1. *Miller, K. S., and S. G. Samko. "Completely monotonic functions." Integral Transforms and Special Functions 12, no. 4 (December 2001): 389–402.*
2. *CLARK, W. EDWIN, and MOURAD E. H. ISMAIL. "INEQUALITIES INVOLVING GAMMA AND PSI FUNCTIONS." Analysis and Applications 01, no. 01 (January 2003):*
3. *Rubinov, A. M. "Monotonic analysis: convergence of sequences of monotone functions." Optimization 52, no. 6 (December 2003): 673–92.*
4. *Guo, Senlin. "Some properties of functions related to completely monotonic functions."*
5. *Gale, Abraham, and Amélie Marian. "Explaining monotonic ranking functions." Proceedings of the VLDB Endowment 14, no. 4 (December 2020)*

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