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SPECTRAL MATRIX METHODS FOR SOCIAL INFLUENCE AND COMMUNITY FORMATION IN COMPLEX NETWORKS

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Abstract:

The analysis of social networks has become an important interdisciplinary re-search area involving mathematics, sociology, and computer science. Graph theo-retic models provide useful tools for representing relationships among individuals, yet classical matrix representations often fail to incorporate variations in influence strength within networks. In this paper, we introduce a matrix-based framework called the Social Influence Spectral Matrix (SISM). The proposed matrix integrates network connectivity, in-fluence weights, and degree-based reinforcement into a unified spectral model. We also introduce a new measure called the Spectral Social Cohesion Index (SSCI) to quantify the strength of community formation in a network.

At the present stage, the study should be viewed as a proposed framework accompanied by preliminary theoretical observations. The results presented here provide motivation for further mathematical investigation and empirical validation in the study of influence dynamics in social networks.

Introduction:

Social networks play a fundamental role in modern society. Individuals interact through communication, collaboration, and social influence, forming complex systems that can be analyzed mathematically. Graph theory provides a natural framework for modeling these interactions where vertices represent individuals and edges represent relationships.

Matrix representations of graphs allow the application of linear algebraic techniques to analyze structural properties of networks. The adjacency matrix and Laplacian matrix are widely used to study connectivity, clustering, and diffusion processes within networks. However, many real-world social systems involve interactions where the strength of influence varies significantly between individuals. Traditional adjacency matrices treat connections as binary relationships and therefore may not fully capture the dynamics of influence propagation.

Motivated by this limitation, this paper proposes a new matrix formulation called the Social Influence Spectral Matrix. The goal of this framework is to combine structural connectivity and influence intensity within a unified spectral representation.

Mathematical Preliminaries:

Let G = (V, E) be a graph representing a social network where V is the set of vertices and E is the set of edges.

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1. **Adjacency Matrix**

   The adjacency matrix A = (a_{ij}) is defined as

$$a_{ij} = \begin{cases} 1 & \text{if vertices } i \text{ and } j \text{ are connected} \\ 0 & \text{otherwise} \end{cases}$$

2. **Degree Matrix**

   The degree matrix D is defined as

$$D = diag(d_1, d_2, \ldots, d_n)$$

   where di is the degree of vertex i.

3 **Social Influence Spectral Matrix**

   Definition 1. Let A be the adjacency matrix of a graph G, W be a matrix representing influence weights between individuals, and D be the degree matrix.

   The Social Influence Spectral Matrix is defined as

$$S = \alpha A + \beta W + \gamma D$$

   where α, β, γ ≥ 0 are parameters controlling the contribution of each component.

   This matrix incorporates structural connectivity, influence strength, and reinforce-ment effects within a single spectral representation.

4 **Influence Propagation Model**

   Let xt represent a vector describing influence levels of individuals at time t.

   Influence propagation may be modeled using the linear system

$$x_{t+1} = Sx_t$$

   This model provides a simple mechanism for studying how influence spreads through a network over time.

Illustrative Matrix Example:

Consider a network of four individuals with adjacency matrix

$$A = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

corresponding degree matrix is

$$D = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Using chosen parameters *α, β, γ*, we may construct the Social Influence Spectral Ma-trix.

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Preliminary Theoretical Observations:

Proposition 1. *Consider the influence propagation model*

$x_{t+1} = Sx_t$

If the spectral radius $\rho(S) < 1$, *then the system converges to equilibrium.*

Proof. The solution of the system is

$x_t = S^t x_0$

If $\rho(S) < 1$, then $S^t \to 0$ as $t \to \infty$, which implies $x_t \to 0$. Therefore the system is stable.

These results represent preliminary theoretical observations rather than complete characterization of influence dynamics.

Spectral Social Cohesion Index:

Definition 2. *The Spectral Social Cohesion Index is defined as*

$$SSCI(G) = \frac{\lambda_{max}(S)}{\sum_{i=1}^{n} d_i}$$

This measure provides an initial indicator of the level of cohesion within a network.

Discussion:

The proposed matrix formulation provides a flexible framework for modeling social influ-ence. Unlike traditional adjacency matrices, the Social Influence Spectral Matrix incor-porates influence intensity and structural reinforcement.

Further research is required to investigate the spectral properties of this matrix in greater detail and to explore its relationship with existing methods such as spectral clustering and modularity-based community detection.

Conclusion:

This paper introduced the Social Influence Spectral Matrix as a new conceptual frame-work for studying influence propagation in social networks. The proposed Spectral Social Cohesion Index provides an initial measure for evaluating network cohesion.

At its current stage, the study should be viewed as a proposed framework accompanied by preliminary theoretical observations. Future work will focus on rigorous theoretical development, empirical validation using real-world social networks, and algorithmic ap-plications.

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